Comparative Study of Polar Codes with QPSK and High-Order QAM in AWGN Channels

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ABSTRACT

This work provides a comprehensive evaluation of communication systems employing quadrature phase-shift keying (QPSK) and high-order quadrature amplitude modulation (QAM) schemes (16-QAM to 2048-QAM) over additive white Gaussian noise (AWGN) channels. The study compares uncoded transmission with polar-coded systems to assess error-correction efficacy. Important measurements like bit error rate (BER) receive thorough evaluations via comprehensive simulations. Without polar codes, high-order QAM (e.g., 2048-QAM) exhibits significantly higher BER, particularly at low signal-to-noise ratios (SNRs), due to increased noise susceptibility. In contrast, QPSK maintains robustness owing to its lower information rate. Monte Carlo simulations are conducted to measure BER performance across modulation schemes, both with and without polar codes. The polar code construction employs successive cancellation decoding, with code rates optimized for each modulation order. Channel conditions are modeled using AWGN with varying signal-to-noise ratios (SNRs). Polar codes markedly improve error resilience across all modulation schemes: QPSK achieves near-error-free performance, while high-order QAM schemes show substantial BER reductions. However, polar codes' effectiveness diminishes with higher modulation orders. For example, 2048-QAM requires significantly greater computational effort for marginal gains in spectral efficiency. The study highlights a critical trade-off: uncoded high-throughput QAM systems achieve higher data rates but suffer from elevated BER, whereas polar-coded systems prioritize reliability at the expense of throughput. Practical recommendations are provided for selecting modulation-code pairings tailored to channel conditions. These insights are vital for designing adaptive communication systems that balance data rate requirements with error-correction capabilities in AWGN environments.

KEYWORDS: Polar codes, QPSK, high-order QAM, AWGN, BER

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INTRODUCTION

Digital communication is utilized for communications that are fundamentally analog and continuous-time, like speech and images, as well as for signals that are fundamentally digital, such as text files [1]. Digital modulation transforms the baseband signal into a digital signal. It is favored over analog modulation for the following reasons: The benefits include enhanced noise immunity and resilience to channel impairments, as well as the integration of digital error correction codes for the detection and rectification of transmission errors. The fundamental digital modulation techniques include Frequency Shift Keying, Phase Shift Keying, BPSK, QPSK, and QAM. The selection of a specific digital modulation scheme is influenced by quality factors including the ability to maintain low bit error rates at low received signal-to-noise ratios, effective performance in the presence of interference, multipath, and fading environments, bandwidth utilization, and ease of implementation at a reasonable cost [2-4]. Polar codes, introduced by Arikan, represent a fundamental category of error-correcting codes that can demonstrably attain the capacity of a binary discrete memoryless channel (B-DMC) as the code length approaches infinity. Polar codes are decoded recursively through the successive cancellation list (SC) algorithm. The successive cancellation list (SCL) presented in [6] demonstrates notable performance enhancement relative to SC decoding. In a SCL decoder, both 0 and 1 are treated as estimated bits, resulting in the generation of two decoding paths at each stage of the decoding process. The cyclic redundancy check (CRC) is employed in [7, 8] to determine the appropriate decoding path in the SCL algorithm. Nonetheless, SCL decoding exhibits significantly greater decoding complexity [9, 10] in comparison to SC. This paper presents a novel decoding approach for polar codes, specifically addressing the scenario where LR=1. The proposed method utilizes parallel multi-SC decoders with improved decision functions to rectify lost bits, thereby resulting in an increase in error propagation. The proposed technique offers a flexible configuration and facilitates the pruning of unnecessary path searching operations, thereby reducing decoding complexity. Multi-Parallel SC decoding demonstrates a notable enhancement in performance relative to traditional SC decoding. This study examines the efficacy of digital modulation schemes, such as QPSK and M-QAM, in two scenarios: in the absence of polar codes and in their presence. It examines how these factors affect the Bit Error Rate (BER) to identify the most reliable system. The research examines Bit Error Rate (BER) in uncoded systems by comparing several M-ary modulation techniques. It examines the influence of varying signal-to-noise ratios (SNR) on bit error rate (BER) and graphically evaluates the correlations between SNR-BER and BER-M-ary values, emphasizing the trade-offs between spectral efficiency and error tolerance. The research examines the effects of integrating polar codes with modulation techniques in polar-coded systems. It emphasizes enhancing Bit Error Rate (BER) performance and assessing coding improvements and error resilience across various Signal-to-Noise Ratio (SNR) circumstances. Graphical analyses examine the disparities between polar-coded and uncoded SNR-BER trends, assess the impact of M-ary numbers on coded systems, and evaluate the efficacy of polar codes in reducing BER across various modulation patterns. The uncoded scenario solely examines fundamental modulation efficacy. The polar-coded case illustrates the interplay between channel coding and modulation, demonstrating the efficacy of adaptive error correction and enhanced robustness in high-noise or high-order modulation scenarios.

Preliminaries and Notations

This study considers binary discrete memoryless channels (B-DMC) for demonstrative purposes. In this study, we describe a generic binary discrete memoryless channel (B-DMC) as W:X→Y, where X represents the input alphabet, Y the output alphabet, and $W(y|x)$ signifies the transition probabilities for $x∈X$ and $y∈Y$. In a BEC, the input alphabet X is selected from the binary set $\{0, 1\}$, whereas Y and the transition probabilities can assume any values. In the manuscript, y_1^N=(y_1,y_2,…,y_N) represents the observations of the code bits x_1^N=(x_1,x_2,…,x_N), which are derived from the encoding of the information bits u_1^N=(u_1,u_2,…,u_N) through N copies of the channel W**.**

Successive Cancellation Decoding

Originally proposed by Erdal Arikan in 2009, Successive Cancellation Decoding (SCD) is a basic decoding method designed for Polar Codes. Developed to reach capacity for symmetric binary-input memoryless channels, Polar Codes show a significant development in coding theory. Channel polarization is the foundation of polar coding: it turns a group of identical independent channels into a set of "polarized" channels where some channels show low dependability while others show great dependability. Executing a series of binary decisions for every bit in the transmitted codeword, the Successive Cancellation Decoding algorithm codes the entire sequence sequentially. This method uses Polar Codes' architecture to enable efficient resource use in hardware implementation as well as decoding [11].

Although the basic SCD method is simple, different methods that increase error-correcting effectiveness and reduce latency can help to enhance it. Over the years, several enhancements to the basic SCD have been proposed-such as CRC-aided decoding and Successive Cancellation List Decoding (SCL)-which significantly increase the efficiency of Polar Codes by incorporating additional information from Cyclic Redundancy Checks (CRCs) [12].At the destination, the received word $y_1^N = (y_1, y_2, \dots, y_N)$ is utilized to estimate information bits successively through the likelihood ratios (LRs) associated with the bits in the code structure. This paper employs a binary erasure channel for performance evaluation. A bit transmitted via a binary erasure channel is received correctly with a probability of $1-\epsilon$, while it is lost with a probability of ϵ. It is demonstrated in equation 1 that the likelihood ratios of the information bits can be expressed recursively and W is a BEC.

- Log-likelihood ratio (LLR) where

$$L_N^{(i)}(y_1^N, \hat{u}_1^{i-1}) = \ln\left(\frac{W_N^{(i)}((y_1^N, \hat{u}_1^{i-1} | u_i = 0))}{W_N^{(i)}((y_1^N, \hat{u}_1^{i-1} | u_i = 1)}\right) \quad (1)$$

- likelihood ratio (LR) where

$$LR(y_1^N, \hat{u}_1^{i-1}) = \ln\left(\frac{W_N^{(i)}((y_1^N, \hat{u}_1^{i-1} | u_i = 0))}{W_N^{(i)}((y_1^N, \hat{u}_1^{i-1} | u_i = 1)}\right) \quad (2)$$

The decision is determined based on

$$\hat{u}_i = \begin{cases} 0 & \text{if } LR(\hat{u}_i) \geq 1 \\ 1 & \text{otherwise} \end{cases} \quad (3)$$

Where $LR(\hat{u}_i)$ is defined as

$$LR(\hat{u}_i) = \frac{prob(\hat{u}_i = 0 | y)}{prob(\hat{u}_i = 1 | y)} \quad (4)$$

2. Simulation Program Description

The error-correction performance of Polar codes across QPSK, 16QAM, 64QAM, 512QAM, 1024QAM, and 2048QAM modulations in Additive White Gaussian Noise (AWGN) channels is evaluated in this work. The durability of 5G New Radio (NR)-compliant Polar coding in noisy environments is evaluated by the simulation measuring the Bit Error Rate (BER) as a function of Signal-to- Noise Ratio (SNR).

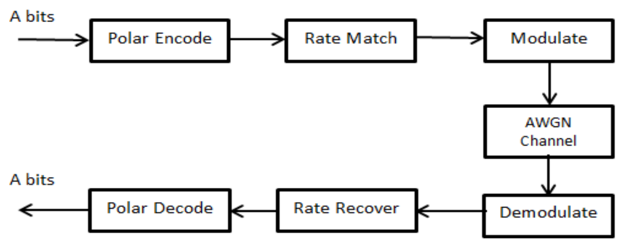

Fig. (1). Block diagram of communication system with Polar code.

SYSTEM PARAMETERS

The efficacy of Polar codes across various SNR levels is statistically evaluated by Monte Carlo methods within the simulation environment. Essential components include:

-Code Configuration: A (E, K) Polar coding is utilized, with E=256 indicating the codeword length and K=132 representing the number of information bits. Improved reliability is attained with a successive cancellation list (SCL) decoder featuring a list size of L=8.
-Analyzed within the range of −5 to 40 dB SNR in increments of 1 dB.

-Each SNR point conducts N=1000 independent tests to ensure the convergence of BER estimates.
-The modulated symbols are transmitted over an AWGN channel, yielding received signals:

$$y = x + n \quad (5)$$

- Demodulation: Log-Likelihood Ratios (LLRs) are computed via soft-output QPSK, 16QAM, 64QAM, 512QAM, 1024QAM, and 2048QAM modulations

$$LLR(b_i) = \log\frac{P(b_i = 0 | y)}{P(b_i = 1 | y)} \quad (6)$$

-Polar Decoding: The SCL decoder processes the LLRs to estimate the transmitted message $\hat{u}$.

-The BER is calculated as:

$$BER = \frac{1}{N.K}\sum_{i=1}^{N}\sum_{j=1}^{K}|u_j - u_i| \quad (7)$$

RESULTS

COMMUNCATION SYSTEM WITHOUT POLAR CODE

The Signal-to-Noise Ratio (SNR) versus Bit Error Rate (BER) performance for communication systems employing QPSK, 16QAM, 64QAM, 512QAM, 1024QAM, and 2048QAM modulation schemes in Additive White Gaussian Noise (AWGN) channels—

without polar coding—is depicted in Figures 2. The results demonstrate that the proposed scheme achieves significantly improved BER performance compared to conventional configurations.

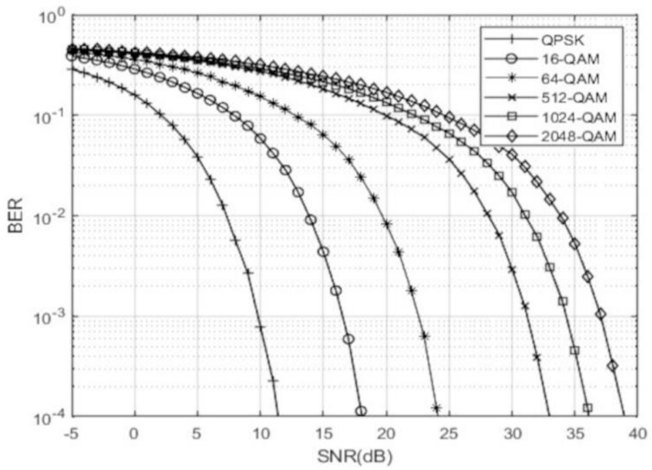

Fig. (2). BER VS SNR plots for communication system without Polar code

The comparison of the performances in terms of bit-error rate (BER) among the various modulations described in Figure 2 - QPSK, 16-QAM, 64-QAM, 128-QAM, 512-QAM, 1024-QAM, and 2048-QAM-indicates that, under the same SNR environments, they offer fairly different error tolerances. From the results, the QPSK modulation achieves a BER of 0.0007 with an SNR of 10 dB, while 2048-QAM with the same SNR gives a much higher BER of 0.319. This difference represents QPSK as a markedly superior candidate under noisy channels compared with the high-order 2048-QAM scheme.
These results pinpoint an important trade-off in digital communication systems: 2048-QAM allows for higher data rates, whereas QPSK is much more reliable when minimizing transmission errors is of prime importance. Therefore, the optimal choice of a modulation scheme rests squarely on the demand posed by the system, weighing spectral efficiency against error performance.

Table (1) illustrates the performance improvement of a system operating without Polar coding under AWGN environments.

Table 1. BER values for modulation schemes without Polar code

Bit Error Rate (BER)							
SNR	QPSK	16-QAM	64-QAM	128-QAM	512-QAM	1024-QAM	2048-QAM
-5	0.287	0.426	0.456	0.546	0.546	0.546	0.546
0	0.159	0.2881	0.358	0.423	0.422	0.423	0.424
5	0.0038	0.1649	0.262	0.3783	0.378	0.374	0.378
10	0.0007	0.0585	0.154	0.2926	0.319	0.312	0.319
15	0	0.0043	0.0642	0.183	0.183	0.215	0.243
20	0	0	0.0082	0.0988	0.0988	0.1337	0.169
25	0	0	0	0.0363	0.0363	0.0663	0.094
30	0	0	0	0.0029	0.0029	0.017	0.0408
35	0	0	0	0	0	0.0004	0.0052
40	0	0	0	0	0	0	0

COMMUNCATION SYSTEM WITH POLAR CODE

The results highlight polar codes' capacity to mitigate the BER degradation inherent in high-order QAM. By exploiting channel polarization, polar codes protect information bits through frozen bit allocation, effectively countering AWGN-induced errors. The greater coding gain for higher QAM aligns with their steeper BER-SNR curves, where marginal SNR improvements yield substantial BER reductions. However, polar codes introduce latency and complexity proportional to block length, necessitating design trade-offs between throughput, power, and computational resources. Compared to system without polar code, polar codes offer competitive performance at lower SNR, particularly in high-order modulation regimes.

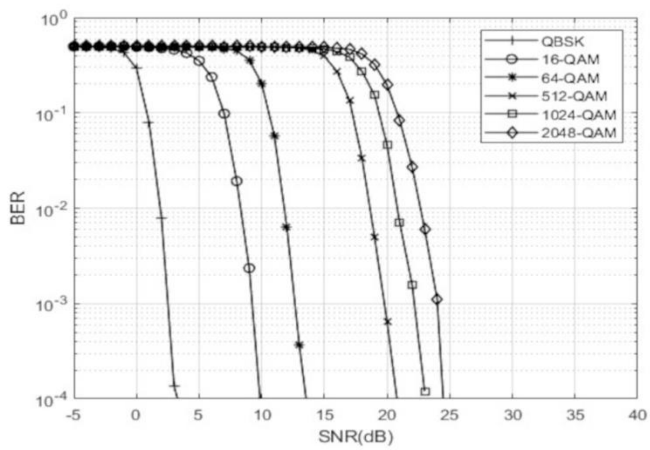

Fig. (3). BER VS SNR plots for communication system without Polar code

Figure 3 will show a comparative study of BER performance between QPSK and other higher-order modulation schemes, including 16-QAM, 64-QAM, 128-QAM, 512-QAM, 1024-QAM, 2048-QAM, with coding in AWGN environments. The results clearly demonstrated the fact that, as opposed to uncoded systems, the systems that use Polar codes have better error performance than those that have no code. The Polar-coded curves noticeably show less flattening at high SNR values, indicating an effective decoding and faster reduction of BER with increasing SNR.

Thus, improvement in the BER performance using Polar coding has been experienced for all modulation orders. For example, it was observed that QPSK with Polar coding can bring out a near error-free (BER ≈ 0) transmission at an SNR of 10 decibels. But at this SNR, the performance of 2048-QAM is measured to be 0.439.

Table 2. BER values for modulation schemes with Polar code

Bit Error Rate (BER)							
SNR	**QPSK**	**16-QAM**	**64-QAM**	**128-QAM**	**512-QAM**	**1024-QAM**	**2048-QAM**
-5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0	0.2913	0.498	0.498	0.498	0.498	0.498	0.498
5	0	0.350	0.2009	0.498	0.498	0.498	0.498
10	0	0	0	0.493	0.495	0.493	0.4937
15	0	0	0	0.390	0.3909	0.487	0.4678
20	0	0	0	0	0.0006	0.0458	0.1951
25	0	0	0	0	0	0	0

Table (2) illustrates the performance improvement of a system operating with Polar coding under AWGN environments.

DISCUSSION

This work compares polar codes in AWGN channels using QPSK and high-order QAM holistically. The main concentration of this work is the complex interaction between SNR performance and BER. Based on the modulation method used, polar codes show different degrees of efficiency; especially, their capacity-achieving characteristics during consecutive cancellation decoding are seen. By means of an integrated QPSK system, the system produces a continuous envelope structure and a reduced modulation order, improving noise immunity. This lowers BER in cases of moderate SNR; hence, QPSK shows benefits in demanding channel environments or when energy resources are limited. On the other hand, because it sends several bits per symbol, hence improving spectral efficiency, high-order QAM provides a denser constellation more sensitive to AWGN. Systems using high-order QAM show better BER at reduced SNR levels than those using QPSK. Increasing SNR greatly reduces the BER difference between QPSK and high-order QAM. This suggests that polar codes can reasonably reduce noise in high-order QAM under certain channel circumstances. This emphasizes a basic trade-off: whereas QPSK shows strong error performance in noisy situations, high-order QAM offers great throughput under suitable channel circumstances. Based on real-time SNR data, the work highlights the possibilities of adaptive modulation schemes that can dynamically transition between QPSK and high-order QAM. These methods help communication systems to maximize dependability and data rate, thereby enhancing performance under different running environments. By means of decoding techniques such as list decoding or belief propagation, one can improve the error-correcting capacity of polar codes, hence lowering the performance gap at lower SNRs and so increasing the efficacy of high-order QAM systems in noisy situations. Simulation results confirm theoretical predictions that the spectral efficiency increases of high-order QAM gain significance with improved channel conditions; QPSK shows strong advantages in low-SNR settings. This dual performance quality provides important direction for the evolution of next-generation communication systems in applications defined by dynamic fluctuations in channel circumstances. The relationships between modulation and coding are clarified by comparison of polar codes with QPSK and high-order QAM in AWGN channels. This approach helps to build adaptive transmission methods that maximize spectral efficiency while reducing error rates, hence improving the resilience and efficiency of wireless communication systems in varied and complicated channel conditions.

CONCLUSION

This research looked at the performance of Polar codes with QPSK and high-order QAM schemes (from 16-QAM to 2048-QAM) in AWGN channels, with a focus on BER and decoding complexity. Results show that there is a clear trade-off between modulation order and error resilience: high-order QAM (e.g., 2048-QAM) achieves higher data rates but has far worse BER performance because it is more noise sensitive. At the other end of the spectrum, QPSK has robust error tolerance but lower throughput. While polar codes do a great job of improving reliability with lots of different modulation schemes, their error-correction performance gets worse as the QAM density gets higher, particularly in really noisy settings. Based on these results, it is clear that modulation-coding methods must be adjusted to fit the channel quality and application requirements. In low-SNR situations where dependability is paramount, QPSK is preferred over high-order QAM with Polar codes, which are better suited to high-SNR environments that demand spectral efficiency. This research lays the groundwork for future work in AWGN channel optimization of communication systems by bringing together issues of operational constraints, code design, and modulation complexity.

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CONSENT FOR PUBLICATION

I hereby provide consent for the publication of the manuscript detailed above, including any accompanying images or data contained within the manuscript that may directly or indirectly disclose my identity

AUTHOR'S CONTRIBUTION

The contents of this manuscript are entirely my responsibility, and I provide my explicit approval to submit it. The results have been carefully examined, and I have given my approval to the final manuscript version.

AVAILABILITY OF DATA AND MATERIALS

The article contains the relevant data and supporting information.

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The author declared no potential conflicts of interest with respect to the research.