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Accuracy Comparison between F-RLS algorithm for CARAR systems and F-RLS algorithm for CARARMA systems

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ABSTRACT

In this paper, the data filtering based recursive least squares algorithm for a CARAR systems, and data filtering based recursive least squares algorithm for a CARARMA systems are derived for comparison. These algorithms are based on the decomposition technique and in this technique, the main algorithm transform into two sub algorithms with smaller sizes. First, System identification model and another is the noise identification model. The problem here is the unknown variables in the information vectors and the used idea for solving this problem is to replacing these unknown variables with their corresponding estimates. Thus, the parameters of these two identification models can be estimated using recursive least squares method. Finally, a simulation example is provided to support the comparison between these proposed algorithms.

KEYWORDS: identification algorithm; parameter estimation; filtering technique; decomposition technique; information vector.

INTRODUCTION

System identification has been used in many fields, including system analysis and process control, and is an essential technique for establishing regular mathematical models from the gathering of measured data and prior knowledge [1]. Approaches for parameter estimation in system identification have attracted a lot of interest. With these techniques, the parameters are estimated using actual data [2]. One of the most popular methods in parameter estimation approach is least squares (LS) method [1]. There are two classes of LS methods. Iterative identification methods, of the first class, are applicable for offline identification; recursive identification methods, of the second class, are applicable for online identification [3]. RLS method can be employed only for Auto –Regressive model (ARX), or equation model, where . RLS approach cannot be utilized for other models, such as the autoregressive moving average model (ARMAX), which means that, or the controlled autoregressive autoregressive moving average (CARARMA) model. The filtering technique is among the most significant methods for dealing with these kinds of models. The basic concept of this technique is to decompose a main system into two filtered sub-system, a filtered system as well as a filtered noise system. As a result, the covariance matrices dimensions become small and computational efficiency becomes high [4]. The rest of the paper is structured as: Section 2 presents filtering based recursive least squares algorithm for a CARAR systems. Section 3, proposes filtering based recursive least squares algorithm for a CARARMA systems. Section 4, results of the simulation are displayed. Finally, Section 5 provides conclusions.

FILTERED DATA BASED RECURSIVE LEAST SQUARES ALGORITHM FOR CARAR MODEL

Consider the following controlled autoregressive autoregressive model illustrated in Fig. 1,

$$A(z)y(k) = B(z)u(k) + \frac{1}{C(z)}v(k) \tag{1}$$

Where

$u(k) = [u(1), u(2), ..., u(n_b)]$ is the system input vector, $y(k) = [y(1), y(2), ..., y(n_a)]$ is the system output vector and $n(k) = [n(1), n(2), ..., n(n_c)]$ is the noise vector. $A(z)$, $B(z)$ and $C(z)$ indicate the polynomials in the unit backward shift operator z^{-1} $[i.e., z^{-1}y(k) = y(k-1)]$ [5], and defined by

$$\begin{aligned} A(z) &= 1 + a_1 z^{-1} + ... + a_{n_a} z^{-na} \\ B(z) &= b_1 z^{-1} + ... + b_{n_b} z^{-nb} \\ C(z) &= 1 + c_1 z^{-1} + ... + c_{nc} z^{-nc} \end{aligned} \tag{2}$$

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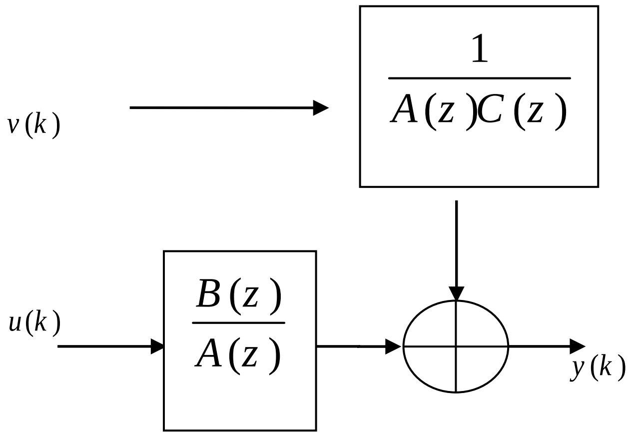


The system depicted by CARAR model

System identification model is described as follows: $y_s(k) = -\sum_{i=1}^{n} a_k y(k-i) + \sum_{i=1}^{n} b_k u(k-i)$ (3)

Additionally, it can be expressed as

$$y_s(k) = \phi_s^T(k)\theta_s \quad (4)$$

Where,

$$\phi_s^T(k) = [-y(k-1)\ldots-y(k-n), u(k-1)\ldots u(k-n)]$$

and

$$\theta_s = [a_1, a_2, \ldots a_n, b_1, b_2, \ldots, b_n]^T$$

are the system information vector and the system parameter vector respectively. The noise identification model is also described as

$$v(k) = \frac{1}{C(z^{-1})} n(k) \quad (5)$$

$$or \quad v(k) = -\sum_{i=1}^{nc} c_k v(k-i) + n(k) \quad (6)$$

This equation can be written as

$$v(k) = \phi_n^T(k)\theta_n + n(k) \quad (7)$$

where,

$$\phi_n^T(k) = [-v(k-1)\ldots-v(k-n)]$$

and

$$\theta_n = [c_1, c_2 \ldots, c_n]^T.$$

By substituting (5) and (7) into (2), and the result is

$$y(k) = \phi_s^T\theta_s + \phi_n^T(k)\theta_n + n(k) = \phi^T\theta + n(k) \quad (8)$$

Where,

$$\phi^T = \begin{bmatrix}\phi_s^T & \phi_n^T\end{bmatrix}$$

and

$$\theta = \begin{bmatrix}\theta_s \\ \theta_n\end{bmatrix}$$

Because $\phi_n(k)$ in $\phi^T(k)$ on equation (8) contains immeasurable noise $v(k-i)$, θ cannot be estimated and therefore, the solution is to replace $\phi_n(k)$ and $v(k-i)$ are With $\hat{\phi}_n(k)$ and $\hat{v}(k-i)$ [6].

$$\hat{\phi}_n^T(k) = [-\hat{v}(k-1)\ldots-\hat{v}(k-n)] \quad (9)$$

The objective of this study, the parameters of the system are estimated from input-output sequences using filtered data approach based recursive least squares algorithm. If $C(z)$ is utilized to the input-output data, the RLS algorithm can be used. Because $C(z)$ is unknown, $\hat{C}(z)$ is used to filter the input–output data. The filtered $u_f(k)$, $y_f(k)$ and $\phi_f(k)$ are defined as

$$u_f(k) = C(z)u(k) \quad (10)$$

$$\phi_f^T(k) = [-y_f(k-1) \; -y_f(k-2)\ldots-y_f(k-n), \; u_f(k-1) \; u_f(k-2)\ldots u_f(k-n)] \quad (11)$$

Multiplying both sides of (1) by $C(z)$ gives

The result of multiplying equation (1) by $C(z)$ on both sides becomes

$$A(z)C(z)y(k) = B(z)C(z)u(k) + n(k) \quad (12)$$

This equation can be expressed as

$$A(z)y_f(k) = B(z)u_f(k) + n(k) \quad (13)$$

It is possible to rewrite this filtered model in vector form as [7].

$$y_f(k) = [1 - A(z)]y_f(k) + B(z)u_f(k) + n(k)$$
$$= \phi_f^T\theta + n(k)$$

(14)

$\hat{\theta}_s$ and $\hat{\theta}_n$ able to be calculated using the following equations

$$\hat{\theta}_s(k) = \hat{\theta}_s(k-1) + \gamma_f(k)[y_f(k) - \phi_f^T(k)\hat{\theta}_s(k-1)] \quad (15)$$

$$\gamma_f(k) = P_f(k)\phi_f(k) = [\phi_f^T(k)P_f(k-1)\phi_f(k)+1]^{-1}\phi_f^T(k)P_f(k-1) \quad (16)$$

$$P_f(k) = [I - \gamma_f(k)\phi_f^T(k)]P_f(k-1) \quad (17)$$

$$P_f(0) = \alpha I \quad and, (\alpha = 100, \ldots, 1000) \quad \theta(0) = 0 \quad (18)$$

$$\hat{\theta}_n(k) = \hat{\theta}_n(k-1) + \gamma_n(k)[v_n(k) - \phi_n^T(k)\hat{\theta}_n(k-1)] \quad (19)$$

$$\gamma_n(k) = P_n(k)\phi_n(k) = [\phi_n^T(k)P_n(k-1)\phi_n(k)+1]^{-1}\phi_n^T(k)P_n(k-1) \quad (20)$$

$$P_n(k) = [I - \gamma_n(k)\phi_n^T(k)]P_n(k-1) \quad (21)$$

$u_f(k)$, $y_f(k)$ in $\phi_f(k)$ and $\phi_n(k)$ are unknown as a result of the unknown $C(z)$ and thus, the algorithms in (15)–(21) are not possible be performed. The solution to obtain filtered data RLS algorithm is to replacing the unknown variables with their estimates as follow [4].

$$v(k) = A(z)y(k) - B(z)u(k) = y(t) - \phi_s^T(k)\theta_s \quad (22)$$

Substituting (6) into (22)

$$y(k) = \phi_s^T(k)\theta_s + v(k) = \phi^T(k)\theta + n(k) \quad (23)$$

θ_s is replaced with $\hat{\theta}_s(k-1)$, and therefore

$\hat{v}(k) = y(t) - \phi_s^T(k)\hat{\theta}_s(k-1)$, and

$\hat{n}(k) = \hat{v}(k) - \hat{\phi}_n^T(k)\hat{\theta}_n(k-1)$.

The estimated parameters of the noise model is

$\hat{\theta}_n(k) = [\hat{c}_1(k), \hat{c}_2(k)..., \hat{c}_{nc}(k)]^T$

and $\hat{u}_f(k) = \hat{C}(z)u(k)$, $\hat{y}_f(k) = \hat{C}(z)y(k)$.

$\hat{u}_f(k)$ and $\hat{y}_f(k)$ can be calculated as:

$$\hat{u}_f(k) = u(k) + \hat{c}_1(k)u(k-1) + c_2(k)u(k-2)... + \hat{c}_{n_c}(k)u(k-n_c) \quad (24)$$

$$\hat{y}_f(k) = y(k) + \hat{c}_1(k)y(k-1) + \hat{c}_2(k)y(k-2)... + \hat{c}_{n_c}(k)y(k-n_c) \quad (25)$$

$$\phi_f^T(k) = [-\hat{y}_f(k-1), -\hat{y}_f(k-2), ..., -\hat{y}_f(k-n), \hat{u}_f(k-1), \hat{u}_f(k-2), ..., \hat{u}_f(k-n)] \quad (26)$$

Finally, $\phi_f(k)$ and $\phi_n(k)$ are replaced with $\hat{\phi}_f(k)$ and $\hat{\phi}_n(k)$, $y_f(k)$ with $\hat{y}_f(k)$, and $v(k)$ with $\hat{v}(k)$, the filtered data based RLS algorithm of $\hat{\theta}_s$ and $\hat{\theta}_n$ for the CARAR system is obtained [8].

$$\hat{\theta}_s(k) = \hat{\theta}_s(k-1) + \gamma_f(k)[\hat{y}_f(k) - \hat{\phi}_f^T(k)\hat{\theta}_s(k-1)] \quad (27)$$

$$\gamma_f(k) = P_f(k)\hat{\phi}_f(k) = [\hat{\phi}_f^T(k)P_f(k-1)\hat{\phi}_f(k) + 1]^{-1}\hat{\phi}_f^T(k)P_f(k-1) \quad (28)$$

$$P_f(k) = [I - \gamma_f(k)\hat{\phi}_f^T(k)]P_f(k-1) \quad (29)$$

$$\hat{\theta}_n(k) = \hat{\theta}_n(k-1) + \gamma_n(k)[\hat{v}_n(k) - \hat{\phi}_n^T(k)\hat{\theta}_n(k-1)] \quad (30)$$

$$\gamma_n(k) = P_n(k)\hat{\phi}_n(k) = [\hat{\phi}_n^T(k)P_n(k-1)\hat{\phi}_n(k) + 1]^{-1}\hat{\phi}_n^T(k)P_n(k-1) \quad (31)$$

$$P_n(k) = [I - \gamma_n(k)\hat{\phi}_n^T(k)]P_n(k-1) \quad (32)$$

$$\hat{v}(k) = y(t) - \phi_s^T(k)\hat{\theta}_s(k-1) \quad (33)$$

$$\hat{\theta}_s(k) = \left[\hat{a}_1(k), \hat{a}_2(k), ...\hat{a}_n(k), \hat{b}_1(k), \hat{b}_2(k), ..., \hat{b}_n(k)\right]^T \quad (34)$$

$$\hat{\theta}_n(k) = [\hat{c}_1(k), \hat{c}_2(k), ..\hat{c}_{nc}(k)]^T \quad (35)$$

FILTERED DATA BASED RECURSIVE LEAST SQUARES ALGORITHM FOR CARARMA MODEL

Consider the following controlled autoregressive autoregressive moving average (CARARMA) model, depicted in Figure 1:

$$A(z)y(k) = B(z)u(k) + \frac{D(z)}{C(z)}n(k) \quad (36)$$

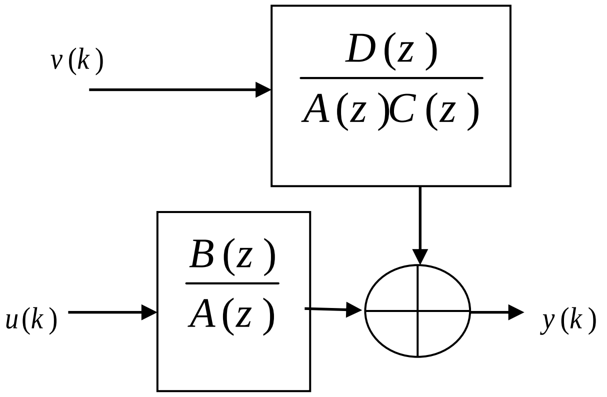
Where $u(k)$ and $y(k)$ are the measured system input and the measured system output respectively, $n(k)$ is stochastic input, or noise. $A(z)$, $B(z)$, $C(z)$ and $D(z)$ are the polynomials and specify as:

$$A(z) = 1 + a_1 z^{-1} + ... + a_{n_a} z^{-na}$$

$$B(z) = b_1 z^{-1} + ... + b_{n_b} z^{-nb}$$

$$C(z) = 1 + c_1 z^{-1} + ... + c_{nc} z^{-nc}$$

$$D(z) = 1 + D_1 z^{-1} + ... + D_{nd} z^{-nd}$$



The system depicted by CARARMA model

In this section, the derivation of identification model in equation (36) is based on decomposition technique that convert this identification model into two small sizes sub models. First one is system identification model and second is noise identification model. A first model is represented by the following equation [6].

$$y_s(k) = -\sum_{i=1}^{n} a_k y(k-i) + \sum_{i=1}^{n} b_k u(k-i) = Z_s^T(k)\theta_s \quad (37)$$

$$\theta_s = [a_1, a_2, ...a_{na}, b_1, b_2, ..., b_{nb}]^T$$

$$Z_s^T(k) = [-y(k-1)... - y(k-na), u(k-1)..u(k-nb)].$$

The second model is

$$v(k) = \frac{D(z^{-1})}{C(z^{-1})}n(k) = -\sum_{i=1}^{nc} c_k v(k-i) + \sum_{i=1}^{nd} d_k n(k-i) + n(k) == Z_n^T(k)\theta_n + n(k) \quad (38)$$

$$Z_n^T(k) = [-v(k-1)... - v(k-nc), n(k-1)..n(k-nd)]$$

$$\theta_n = [c_1, c_2..., c_{nc}, d_1, d_2..., d_{nd}]^T.$$

Now, equation (1) can be written as

$$y(k) = Z_s^T\theta_s + Z_n^T(k)\theta_n + n(k) = Z^T\theta + n(k) \quad (39)$$

Where

$$Z^T = \begin{bmatrix} Z_s^T & Z_n^T \end{bmatrix}$$

and

$$\theta = \begin{bmatrix} \theta_s \\ \theta_n \end{bmatrix}$$

$Z_n(k)$ in $Z^T(k)$ includes unknown variables $v(k-i)$

and $n(k-i)$.the RLS algorithm cannot be used. Replacing $v(k-i)$ and $n(k-i)$ with $\hat{v}(k-i)$ and $\hat{n}(k-i)$ respectively is the solution.

$$\hat{Z}_n^T(k)=[-\hat{v}(k-1)...-\hat{v}(k-nc),\hat{n}(k-1)...\hat{n}(k-nd)] \quad (40)$$

Then $\hat{Z}^T=\left[Z_s^T \quad \hat{Z}_n^T\right]$.

$$\hat{v}(k)=y(k)-Z_s^T(k)\hat{\theta}_s(k) \quad (41)$$

In this part RLS is developed by using the data filtering technique for estimating the system parameters. The rational fraction $\frac{C(z)}{D(z)}$ is utilized and the model represented by equation (36) becomes equation error. $\frac{\hat{C}(z)}{\hat{D}(z)}$ is used to filter the input–output data because $\frac{C(z)}{D(z)}$ is unknown. The identification technique is referred as the data filtering based recursive least squares algorithm (F-RLS) [4].

Now, the filtered $u_f(k)$, $y_f(k)$ and $Z_f(k)$ are described as

$$u_f(k)=\frac{C(z)}{D(z)}u(k),\ y_f(k)=\frac{C(z)}{D(z)}y(k) \quad (42)$$

$$Z_f^T(k)=[-y_f(k-1)...-y_f(k-na),\ u_f(k-1)..u_f(k-nb)] \quad (43)$$

Equation (36) becomes as:

$$A(z)\frac{C(z)}{D(z)}y(k)=B(z)\frac{C(z)}{D(z)}u(k)+n(k) \quad (44)$$

Or

$$A(z)y_f(k)=B(z)u_f(k)+n(k) \quad (45)$$

Then, $y_f(k)$ is written in the following form:

$$y_f(k)=[1-A(z)]y_f(k)+B(z)u_f(k)+n(k) = Z_f^T\theta+n(k) \quad (46)$$

A recursive least squares algorithm for $\hat{\theta}_s$ and $\hat{\theta}_n$ of θ_s and θ_n are obtained using the following equations:

$$\hat{\theta}_s(k)=\hat{\theta}_s(k-1)+\gamma_f(k)[y_f(k)-Z_f^T(k)\hat{\theta}_s(k-1)] \quad (47)$$

Where

$$\gamma_f(k)=P_f(k)Z_f(k)=[Z_f^T(k)P_f(k-1)Z_f(k)+1]^{-1}Z_f^T(k)P_f(k-1) \quad (48)$$

And the matrix $P(k)$

$$P_f(k)=[I-\gamma_f(k)Z_f^T(k)]P_f(k-1) \quad (49)$$

$$P_f(0)=\alpha I,(\alpha=100,...,1000) \quad and \quad \theta(0)=0 \quad (50)$$

$$\hat{\theta}_n(k)=\hat{\theta}_n(k-1)+\gamma_n(k)[v_n(k)-Z_n^T(k)\hat{\theta}_n(k-1)] \quad (51)$$

$$\gamma_n(k)=P_n(k)Z_n(k)=[Z_n^T(k)P_n(k-1)Z_n(k)+1]^{-1}Z_n^T(k)P_n(k-1) \quad (52)$$

$$P_n(k)=[I-\gamma_n(k)Z_n^T(k)]P_n(k-1) \quad (53)$$

Because $C(z)$ and $D(z)$ are unknown, and therefore $u_f(k)$, $y_f(k)$, $Z_f(k)$ and $Z_n(k)$ are unknown. The algorithms in equations (47)–(53) are not possible be performed [7]. The solution to obtain filtered data RLS algorithm is to replacing the unknown variables with their estimates and new identification algorithms is derived as follow:

$$v(k)=A(z)y(k)-B(z)u(k)=y(t)-Z_s^T(k)\theta_s \quad (54)$$

$$y(k)=Z_s^T(k)\theta_s+v(k)=Z^T(k)\theta+n(k) \quad (55)$$

θ_s on equation (54) is replaced with its estimate. $\hat{v}(k)=y(t)-Z_s^T(k)\hat{\theta}_s(k-1)$.

In addition to, $Z_n(k)$ and θ_n are replaced with their estimate $\hat{Z}_n(k)$ and $\hat{\theta}_n(k-1)$, and

$\hat{n}(k)=\hat{v}(k)-\hat{Z}_n^T(k)\hat{\theta}_n(k-1)$.

Then,

$\hat{u}_f(k)=\frac{\hat{C}(z)}{\hat{D}(z)}u(k)$ and

$\hat{y}_f(k)=\frac{\hat{C}(z)}{\hat{D}(z)}y(k)$.

From the previous equations, we can recursively compute $\hat{u}_f(k)$ and $\hat{y}_f(k)$ by the following equations

$$\hat{u}_f(k) = \frac{\hat{C}(z)}{\hat{D}(z)}u(k) = -\hat{d}_1(k)\hat{u}_f(k-1) - \hat{d}_2(k)\hat{u}_f(k-2) - \ldots\hat{d}_{n_d}(k)\hat{u}_f(k-n_d) + u(k) + \hat{c}_1(k)u(k-1) + \hat{c}_2(k)u(k-2)\ldots + \hat{c}_{n_c}(k)u(k-n_c). \quad (56)$$

$$\hat{y}_f(k) = \frac{\hat{C}(z)}{\hat{D}(z)}y(k) = -\hat{d}_1(k)\hat{y}_f(k-1) - \hat{d}_2(k)\hat{y}_f(k-2) - \ldots\hat{d}_{n_d}(k)\hat{y}_f(k-n_d) + y(k) + \hat{c}_1(k)y(k-1) + \hat{c}_2(k)y(k-2)\ldots + \hat{c}_{n_c}(k)y(k-n_c). \quad (57)$$

$$Z_f^T(k) = [-\hat{y}_f(k-1), -\hat{y}_f(k-2), \ldots, -\hat{y}_f(k-na), \hat{u}_f(k-1), \hat{u}_f(k-2), \ldots, \hat{u}_f(k-nb)] \quad (58)$$

After replacing, $Z_f(k)$, $Z_n(k)$, $y_f(k)$ and $v(k)$ are replaced with $\hat{Z}_f(k)$, $\hat{Z}_n(k)$ $\hat{y}_f(k)$ $\hat{v}(k)$ equations (47)–(53), the filtered data based RLS algorithm of $\hat{\theta}_s$ and $\hat{\theta}_n$ for CARARMA system is obtained as follow [4]:

$$\hat{\theta}_s(k) = \hat{\theta}_s(k-1) + \gamma_f(k)[\hat{y}_f(k) - \hat{Z}_f^T(k)\hat{\theta}_s(k-1)] \quad (59)$$

$$\gamma_f(k) = P_f(k)\hat{Z}_f(k) = [\hat{Z}_f^T(k)P_f(k-1)\hat{Z}_f(k) + 1]^{-1}\hat{Z}_f^T(k)P_f(k-1) \quad (60)$$

$$P_f(k) = [I - \gamma_f(k)\hat{Z}_f^T(k)]P_f(k-1) \quad (61)$$

$$\hat{\theta}_n(k) = \hat{\theta}_n(k-1) + \gamma_n(k)[\hat{v}_n(k) - \hat{Z}_n^T(k)\hat{\theta}_n(k-1)] \quad (62)$$

$$\gamma_n(k) = P_n(k)\hat{Z}_n(k) = [\hat{Z}_n^T(k)P_n(k-1)\hat{Z}_n(k) + 1]^{-1}\hat{Z}_n^T(k)P_n(k-1) \quad (63)$$

$$P_n(k) = [I - \gamma_n(k)\hat{Z}_n^T(k)]P_n(k-1) \quad (64)$$

$$\hat{\theta}_s(k) = \left[\hat{a}_1(k), \hat{a}_2(k), \ldots\hat{a}_{na}(k), \hat{b}_1(k), \hat{b}_2(k), \ldots, \hat{b}_{nb}(k)\right]^T \quad (65)$$

$$\hat{\theta}_n(k) = \left[\hat{c}_1(k), \hat{c}_2(k), \ldots\hat{c}_{nc}(k), \hat{d}_1(k), \hat{d}_2(k), \ldots, \hat{d}_{nd}(k)\right]^T \quad (66)$$

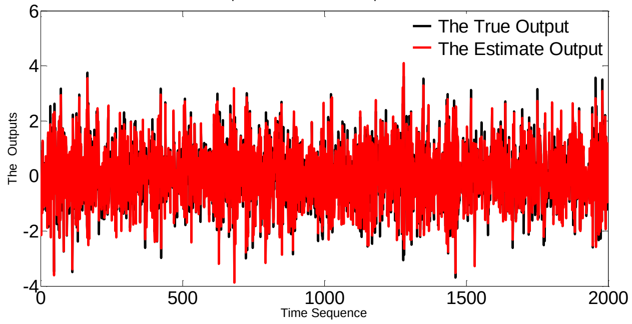
SIMULATION RESULTS

In order to evaluates the effectiveness of the suggested algorithms, the following CARAR and CARARMA systems are considered as:

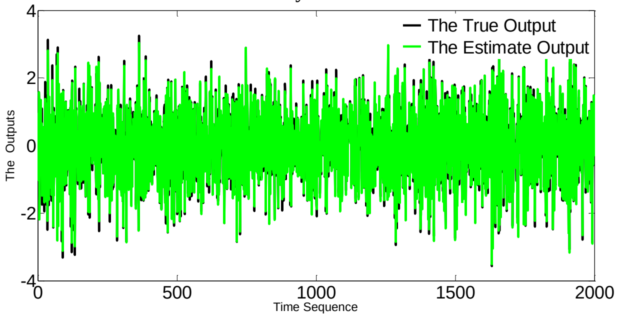
$$y(k) = \frac{0.54z^{-1} + 0.79z^{-2}}{1 + 0.46z^{-1} + 0.61z^{-2}}u(k) + \frac{1}{1 + 0.23z^{-1}}n(k)$$

$$y(k) = \frac{0.54z^{-1} + 0.79z^{-2}}{1 + 0.46z^{-1} + 0.61z^{-2}}u(k) + \frac{1 + 0.31z^{-1}}{1 + 0.45z^{-1}}n(k)$$

$u(n)$ is taken as a white sequence with a Gaussian distribution of zero mean and unit variance, $v(n)$ as sequences of white noise with zero mean and $\sigma^2 = 0.2$. in order to evaluate the performance of these algorithms, the estimated outputs of F-RLS algorithm for CARAR systems versus time sequences is plotted together with it true output. In addition to, the estimated outputs of F-RLS algorithm for CARARMA systems versus time sequences is plotted together with it true output as shown in fig. 3 and fig. 4 respectively.

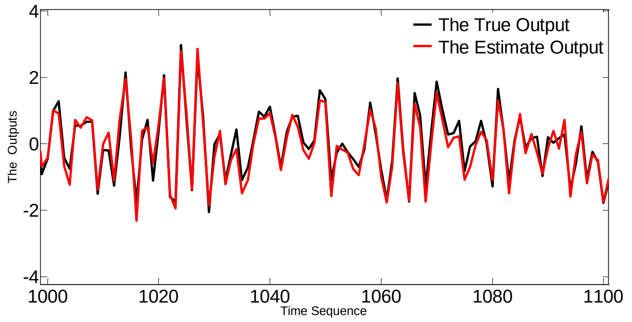


The true output and the Estimated outputs of F-RLS algorithm for CARAR systems

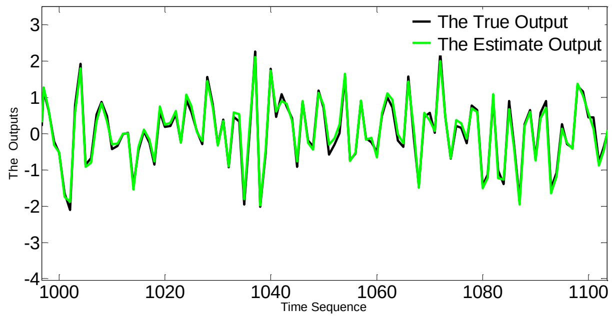


The true output and the Estimated outputs of F-RLS algorithm for CARARMA systems

For more clarification, Window from n =1000 to n =1100 has been taken as offered in Fig. 5 and Fig. 6.



Window from n =1000 to n =1100 of true output and the Estimated outputs of F-RLS algorithm for CARAR systems



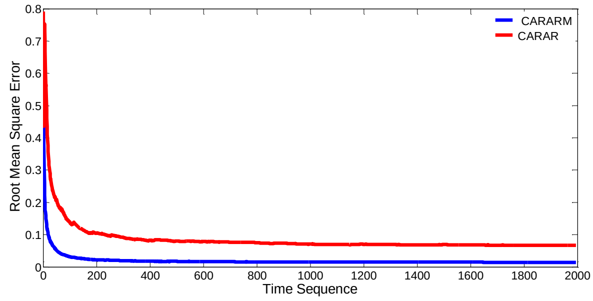
Window from n =1000 to n =1100 of true output and the Estimated outputs of F-RLS algorithm for CARARMA systems

The accuracy of F-RLS algorithm for CARAR systems is compared with the accuracy of F-RLS algorithm for CARARMA systems using root mean square errors (RMSE) method. RMSE is the most common statistical techniques and, it has been extensively utilized to evaluate the model's accuracy. RMSE is ccalculating using the following equation [9]:

$$RMS-Error=\sqrt{\frac{\sum_{i=1}^{N}((\hat{y}_i-y_i))^2}{N}}$$

(67)

The root mean square errors versus time sequences are displayed in fig. 7.



Root-mean-square error of F-RLS algorithm for CARAR (red color) systems and F-RLS algorithm for CARARMA systems (blue color) versus time sequences.

From these figures, the following conclusions have been drawn:

- The F-RLS algorithm for CARAR systems has a low computational load than the F-RLS algorithm for CARARMA systems because the dimension of its covariance matrix of noise subsystem are smaller than that of the covariance in the F-RLS algorithm for CARARMA.
- The estimated output of F-RLS algorithm for CARAR and F-RLS algorithm for CARARMA systems are very close with their true output. This indicate that the performance of these algorithms are very high.
- The root mean square errors become smaller and smaller with the time sequence increasing. This indicates that the proposed algorithms are effective.
- The F-RLS algorithm for CARARMA systems is more accurate than the F-RLS algorithm for CARAR systems. This means that the proposed F-RLS algorithm for CARARMA systems has better identification performance compared with the F-RLS algorithm for CARAR systems.

CONCLUSION

In this paper, the data filtering based recursive least squares algorithm for a CARAR systems is presented, and data filtering based recursive least squares algorithm for a CARARMA systems is proposed for comparison. The F-RLS algorithm for CARAR systems has lower computational burden than the F-RLS algorithm for CARARMA systems, but The F-RLS algorithm for CARARMA systems achieve more accuracy compared with the F-RLS algorithm for CARAR systems. These proposed algorithms of this paper can be extended to study identification problems of other linear systems and nonlinear systems with colored noises.

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