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# Efficiency of the Simplex Method in Solving Transportation Problems

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## ABSTRACT

This study examines the efficiency of the Simplex Method in solving transportation problems related to the Abna Al-Mashat Cement Brick Factory. It focuses on optimizing the transportation of raw materials, such as gravel, from quarries to production units, which is crucial for reducing operational costs. A range of quantitative methods, including the Least Cost Method, Northwest Corner Method, and Vogel's Approximation Method, were employed to compare their effectiveness with the Simplex Method.

The study aims to translate the problem from an economic formulation into a mathematical model, enabling the use of the Simplex Method for efficient resolution. Data on transportation costs, production capacity, and distances between quarries and production centers were analyzed. The results indicated that the Simplex Method provides a cost-effective solution while achieving a balance between supply and demand.

The study highlights the significance of utilizing quantitative methods in transportation decision-making, contributing to improved operational efficiency and cost reduction. It recommends adopting the Simplex Method as a primary tool for transportation management in production enterprises.

**KEYWORDS:** *Linear programming; transportation problems; Simplex Method; Least Cost Method; Northwest Corner Method; Vogel's Approximation Method.*

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## INTRODUCTION

Quantitative methods, including transportation models, assist in decision-making, particularly in operations research. The objective is to find the optimal distribution of goods (such as transporting products from production areas to consumption areas) at the lowest costs. The transportation problem is a crucial tool in quantitative analysis. This study aims to build a transportation problem model as an application of linear programming, assisting decision-makers at the Abna Al-Mashat Cement Brick Factory in making informed decisions regarding the transportation of gravel from quarries to production units by translating hypothetical models into mathematical formulations, utilizing Solver in Excel for resolution. To better understand the context and development of transportation methods, it is essential to explore the existing literature on optimization techniques. Several studies have addressed the use of optimization methods for solving transportation problems, focusing particularly on specialized algorithms such as the transportation simplex, MODI, and stepping-stone methods. However, fewer studies have explored the direct application of the general simplex method to transportation problems. Taha (2017) provides a comprehensive introduction to the simplex method and its theoretical basis in linear programming[9], including transportation models. Hillier and Lieberman (2014) also discuss transportation problems and contrast the performance of the transportation simplex and general simplex approaches[3]. Similarly, Bazaraa et al. (2010) present the simplex method as a foundational tool for solving network flow problems, including transportation systems.[5] In earlier work, Manne (1951) explored the feasibility of using the general simplex method to solve transportation problems, pointing out both its advantages and limitations in terms of computational efficiency[8]. While Dantzig (1998) laid the theoretical groundwork for the simplex method, he emphasized that specialized methods for structured LP problems—like the transportation model—can yield faster solutions.[6] Recent comparative studies, such as those by Arsham and Adlakha (2016), introduced modified simplex-type algorithms tailored for transportation models, demonstrating competitive performance with traditional methods.[4] Despite the abundance of optimization literature, few studies have systematically evaluated the efficiency of the general simplex method when applied directly to classical transportation problems. This paper aims to fill that gap by presenting a detailed analysis of the method's computational performance and practical implications.

## RESEARCH PROBLEM

Companies face challenges in managing transportation costs, which represent a significant portion of total operational expenses. This study aims to analyze how to improve transportation processes using linear programming techniques, focusing on the Simplex Method to solve the transportation problem. The key issue is the need to reduce costs while ensuring that production needs are met, necessitating effective decisions regarding transportation quantities from quarries to production units.

## RESEARCH METHODOLOGY

The study adopts a quantitative data analysis methodology, employing linear programming techniques to develop a mathematical model for the transportation problem. Excel was utilized to apply the Simplex Method, in addition to traditional methods such as the Northwest Corner Method and the Least Cost Method. The results were analyzed to compare the efficiency of various methods and identify which provides the best solutions in terms of transportation costs.

## RESEARCH OBJECTIVE

This research aims to highlight the efficiency of the Simplex Method in solving transportation problems and to provide practical solutions that assist decision-makers in improving transportation cost management. Through data analysis and the application of quantitative methods, the study seeks to offer effective recommendations that contribute to reducing transportation costs and enhancing production efficiency.

## THEORETICAL FRAMEWORK

## DATA DESCRIPTION

This study addresses the transportation problem specific to the Abna Al-Mashat Cement Brick Factory, which faces challenges in transporting raw materials, particularly gravel, from quarries to production units. The data used in the research includes information on transportation costs, production capacity for both factories, and distances between quarries and production centers. Data were collected from local sources, determining the quantities required for each production unit based on daily production capacity.

## LINEAR PROGRAMMING

Linear programming is a mathematical model for allocating a set of limited resources to competing needs under a set of constraints and fixed factors to achieve the best possible outcome. Linear programming models are among the simplest and easiest mathematical models that can be created to address industrial, governmental, and production-related challenges. A linear programming model is formulated by defining an objective function in linear form to obtain a numerical value, seeking to maximize this value if the goal is profit or minimize it if the goal is to reduce costs. Constraints need to be defined as relationships between decision variables and available resources in the form of linear inequalities or equations. Non-negativity conditions must also be established, ensuring that the decision variables are non-negative.[3]

Generally, a linear programming model can be expressed in the following form:

$$Max\ or\ Min \quad Z = C_1x_1 + C_2x_2 + C_3x_3 + \cdots + C_nx_n$$

$S.t$:

$$a_{11}x_1 + a_{12}x_2 + x_{13}x_3 + \cdots + a_{1n}x_n \le, =, \ge b_1$$

$$a_{21}x_1 + a_{22}x_2 + x_{23}x_3 + \cdots + a_{2n}x_n \le, =, \ge b_2$$

$$a_{31}x_1 + a_{32}x_2 + x_{33}x_3 + \cdots + a_{3n}x_n \le, =, \ge b_3$$

. . . .

. . . .

$$a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n \le, =, \ge b_m$$

$$x_1, x_2, x_3, \ldots x_n \ge 0$$

## METHODS FOR SOLVING LINEAR PROGRAMMING

There are two main methods used to find the optimal solution for linear programming models:

1. Graphical Method

The graphical method is commonly used for solving linear models, particularly when there are two variables. However, when more than three variables are present, the graphical method becomes impractical, necessitating the use of another method.

2. Simplex Method

The Simplex Method, invented by George Dantzig in 1947, is a highly efficient mathematical technique for finding optimal solutions to linear programming problems. This method is applicable to linear programming models algebraically, regardless of the number of variables, and is the most widely utilized method for solving these mathematical models [2].

The Simplex Method employs simple mathematical concepts iteratively, meaning the same procedures are repeated until the optimal solution is achieved. It is an iterative approach to analyze linear programming problems, focusing on selecting variables that significantly influence both the objective function and the constraints while disregarding those that do not.

The application of this method can lead to one of three outcomes: a bounded optimal solution, an unbounded optimal solution, or no feasible solution. Initially, the linear programming model must be converted from its primal form to its standard form for the Simplex Method to be applied. Some models may contain constraints expressed as equalities or greater-than-or-equal-to inequalities, which necessitate the introduction of artificial variables. Alternative Simplex methods, such as the Two-Phase Method and the Multi-Stage Simplex Method, are employed depending on the linear model.

## TRANSPORTATION PROBLEMS

The transportation problem model is an application of operations research focused on finding the minimum cost transportation plan between a set of production centers and a set of demand centers. This model was first developed in 1941 by F.L. Hitchcock, who conducted a study titled "Distribution of Production from Several Sources to Several Local Areas." [2] Transportation problems are concerned with the distribution of products from production centers, such as factories, to demand centers, including warehouses and distribution and marketing centers.

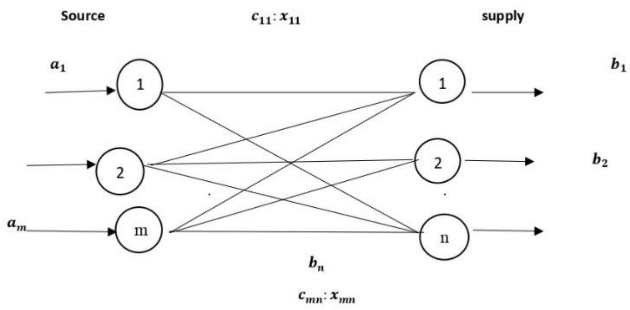


Figure 1: Transfer from source to demand

## GENERAL FORMULATION OF TRASPORTATION PROBLEM MODELS

1. There are (n) production centers and ( m) marketing or demand centers (consumption).
2. The cost of transporting one unit of goods from marketing center iii to demand center ( j )is known and specified as Cij.
3. The quantities transported from the production centers to the specified marketing centers are denoted as Xij
4. The objective is to minimize the total transportation costs. [3]

To facilitate the study of the transportation problem, a cost table can be represented as follows:

Table 1: The costs of the transportation

| Marketing Centers / Production Centers | 1 | 2 | 3 | ... | M | Supply |
|---|---|---|---|---|---|---|
| 1 | $C_{11}$ | $C_{12}$ | $C_{13}$ | ... | $C_{1m}$ | $b_1$ |
| 2 | $C_{21}$ | $C_{22}$ | $C_{23}$ | ... | $C_{2m}$ | $b_2$ |
| 3 | $C_{31}$ | $C_{32}$ | $C_{33}$ | ... | $C_{3m}$ | $b_3$ |
| . . | . . | . . | . . | ... | . . | . . |
| M | $C_{n1}$ | $C_{n2}$ | $C_{n3}$ | ... | $C_{nm}$ | $B_n$ |
| Demand | $a_1$ | $a_2$ | $a_3$ | ... | $A_m$ | |

## METHODS FOR SOLVING TRANSPORTATION PRPBLEMS

### 1. Northwest Corner Method

This method is one of the simplest, as it does not use any scientific approach to distribute the available quantities from sources to meet market needs. The process begins by distributing quantities from the northwest corner (the upper left corner of the table) toward the opposite side until the produced quantities are allocated to the distribution points. The method can be summarized in the following steps:

A. Fill the cell located in the northwest corner X11X_{11}X11 with the minimum quantity ($X_{ij}$) possible, starting with cell $X_{11}$, ensuring the following condition is met:
$X_{ij}=\min(s_i,d_j)$
B. Re-evaluate the supply column and the demand row as follows:
New si = si−Xij , New dj=dj−Xij
C. If si < dj , remove row (i) , If dj < si remove column j from consideration.
D. If si= dj_ remove both row ( i) and column (j) from consideration.
E. Repeat the above steps after modification (removal), starting with the cell located in the northwest corner of the new matrix after deletion [1].

### 2. Least Cost Method

The Least Cost Method is preferred over the Northwest Corner Method because it searches for the lowest cost in the cost matrix and distributes the required quantity according to min (ai,bi) The steps can be summarized as follows:

A. Identify the cell ($x_{ij}$) with the lowest cost in the table (in case of a tie, choose randomly).
B. Fill the selected cell ($X_{ij}$) with the minimum quantity possible, ensuring the following condition is met:
$X_{ij}$=min (si,dj)
C. Re-evaluate the supply column and the demand row as follows:
New $S_i$=si−$X_{ij}$
New $d_j$=dj−$X_{ij}$
D. If si < dj , remove row( i ) from consideration. If dj < $s_i$ remove column( j) from consideration.
E. If si = dj remove both row (i) and column (j) from consideration.
F. Repeat the above steps on the remaining cells until all rows and columns are removed.

### 3. Vogel's Approximation Method

This method can be summarized in the following steps:

A. For each row and column, calculate the penalty cost based on the difference in cost between the two cells with the lowest costs in the row and column.
B. Choose the highest penalty cost from all rows and columns.
C. In the selected row or column, fill the cell $X_{ij}$ with the minimum quantity possible, ensuring the following condition is met:
$X_{ij} = \min(s_i,d_j)$
D. Re-evaluate the supply column and the demand row as follows:
New $s_i = s_i−x_{ij}$ , New dj = $d_j− x_{ij}$
E. If $s_i < d_i$ , remove row (i) from consideration. If $d_j < s_i$ remove column jjj from consideration.
F. If $s_i = d_j$ , remove both row (i) and column ( j) from consideration.
G. Repeat the above steps on the remaining cells until all rows and columns are removed, taking into account the new transportation costs.

## APPLIED STUDY: OVERVIEW OF THE RESEARCH COMMUNITY

**Overview of the Factory**

The focus of this study is on the Abna Al-Mashat Cement Brick Factory (Boumshi), which is owned by a legal entity represented by its general manager. This factory was established in 2012 by the owner, and its primary activity is the production of cement bricks (Boumshi). The factory has two branches of production units:

- **Branch One: Najila - Janzour**
This is the main branch as it contains modern machinery for brick production, with a daily production capacity of 5,000 pieces.
- **Branch Two: Siraj**
This branch has manual machines, resulting in a daily production capacity of 2,500 pieces, which is lower than that of the first branch.

**Raw Materials**

The production of bricks involves several raw materials, including cement, gravel, water, and sand. Thus, we need to transport these raw materials from their sources to the production units. This study will focus on the transportation of gravel from the sources (quarries - crushers) to the production units. Most crushers are located in mountainous areas and produce various types of gravel, including large, medium, and small sizes, depending on the nature of the machinery each crusher has. For cement brick production (Boumshi), we require small-sized gravel (Cornelia).

The factory collaborates with three quarries (crushers) to obtain gravel, which are:

- Al-Aziziyah Crusher
- Al-Rabita Crusher

- Bani Walid Crusher

Gravel is transported to the production units (Najila - Janzour and Siraj) using rented trucks.

## PRESENTATION OF THE TRANSPORTATION PROBLEM AND THE MATHEMATICAL FORMULATION

**Problem Presentation**

In this phase, we will study the problem derived from reality and translate it from its economic form to a mathematical form. This translation is based on several stages and steps that will be followed to obtain the mathematical model of the problem, preparing it for solution.

As previously mentioned, we will present the institution that extracts the raw material to direct it to the production units, as illustrated in the following diagram:

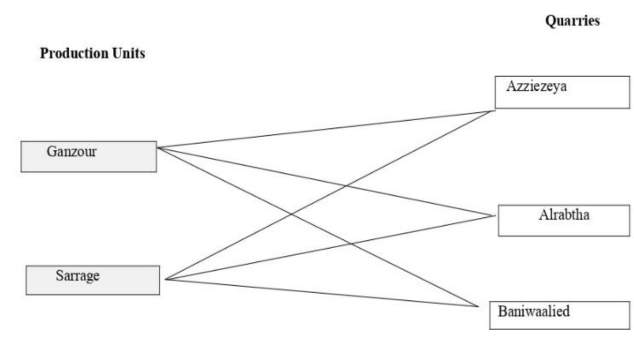


Figure 2. Path identification.

To Meet the Production Units' Demand for Raw Materials

The company bears the cost of transporting raw materials from the quarries to the factories using its own trucks. According to the company's official documents, the cost is calculated based on the distance, measured in kilometers.

Mathematical Formulation

1. The routes have been defined as shown in the previous diagram.
2. The symbols related to the transportation problem are as follows:

Table. 2. Symbols of the Transportation Problem Table

| Production Units / Quarries | Al-Aziziya | Al-Rabita | Bani Walid | Demand |
|---|---|---|---|---|
| First Factory | $C_{11}$ $X_{11}$ | $C_{12}$ $X_{12}$ | $C_{13}$ $X_{13}$ | $A_1$ |
| Second Factory | $C_{21}$ $X_{21}$ | $C_{22}$ $X_{22}$ | $C_{23}$ $X_{23}$ | $A_2$ |
| Supply | $B_1$ | $B_2$ | $B_3$ | |

Symbols and Their Descriptions

- Cij: Cost of transporting one unit from source (i) to destination ( j) (from crushers to factories).
- Xij: Quantities transported from source (i) to destination ( j).
- Am: Quantities offered by the crushers, measured in cubic meters.
- Bm: Quantities required by the production units, measured in cubic meters.

Formulation of the Model for This Problem

$$Min\ Z = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23}$$

S.t

$$x_{11} + x_{12} + x_{13} = A_1$$
$$x_{21} + x_{22} + x_{23} = A_2$$
$$x_{11} + x_{21} = B_1$$
$$x_{12} + x_{22} = B_2$$
$$x_{13} + x_{23} = B_3$$
$$x_{ij} \geq 0$$

## DATA RELATED TO THE TRANSPORTATION PROBLEM (DATA COLLECTION)

The data related to the transportation problem is divided into two types: the first is related to structural constraints, and the second is related to the objective function.

**1- Data Related to Constraints**

There are two sections of constraints: one pertains to the supply of each crusher, which requires knowledge of the quantity produced weekly from natural stone (gravel), and the other pertains to the demand of each production unit for raw materials, which requires knowledge of the quantities needed to fulfill their orders. These orders represent the production capacity of each factory.

Through study and analysis, we concluded that the crushers supply raw materials according to the demand of each production unit. The first factory (Najila - Janzour) requires 35 m³ of gravel and produces 5,000 units of (cement bricks), while the second factory requires 15 m³ of gravel and produces 2,500 m³ daily. The weekly production capacity for each unit is summarized in the following table:

Table 3. Weekly Required Quantities of Raw Materials for Each Production Unit.

| Raw Material Demand (m³) | Quantity Produced (m²) | Factories |
|---|---|---|
| 245م³ | 35000م² | Janzour |
| 105م³ | 17500م² | sarrage |
| 350م³ | 52500م² | Total |

The total quantity required weekly for both factories is 350 m³ of gravel. Since the factories direct their raw material requests to the crushers based on the production capacity of each crusher, the allocation is determined accordingly.

Table 4. Quantities Supplied by Each Crusher for Production

| Crushers | Quantities Directed for Processing (m³) |
|---|---|
| Al-Aziziya | 100 m³ |
| Al-Rabita | 100 m³ |
| Bani Walid | 150 m³ |
| Total | 350 m³ |

Thus, we observe that the total supply equals the total demand, indicating that we are in a balanced state for the transportation table.

## DATA RELATED TO THE OBJECTIVE FUNCTION

The data concerning transportation costs serves as the objective function. The distance from the Al-Aziziyah quarry to the first factory is estimated at 28 km, with a cost of 300 dinars. The transportation cost from the Al-Aziziyah quarry to the second factory is 410 dinars over a distance of 38 km. Since the factory estimated the transportation cost for the load from the Al-Aziziyah quarry to the first factory at 960 dinars, the cost of the load is thus 960 - 300 = 660 dinars. This cost is fixed for all quarries.

Table 5. Distance Between Each Crusher and Production Unit.

| Production Unit | Al-Aziziya | Al-Rabita | Bani Walid |
|---|---|---|---|
| Grass – Janzour | 28 km | 81 km | 173 km |
| Al-Siraj | 38 km | 91 km | 175 km |

1. Transportation Costs by Load According to Distance

Table 6. Transportation Costs by Load According to Distance

| Production Unit | Al-Aziziya | Al-Rabita | Bani Walid |
|---|---|---|---|
| Grass – Janzour | 960 D.L | 1060 D.L | 1210 D.L |
| Al-Siraj | 1070 D.L | 1110 D.L | 1240 D.L |

2. Transportation Costs According to Distance

Table 7. Transportation Costs According to Distance

| Production Units \ Quarries | Al-Aziziya | Al-Rabita | Bani Walid | Demand |
|---|---|---|---|---|
| First Factory | 300 $X_{11}$ | 400 $X_{12}$ | 550 $X_{13}$ | 245 |
| Second Factory | 410 $X_{21}$ | 450 $X_{22}$ | 580 $X_{23}$ | 105 |
| Supply | 100 | 100 | 150 | 350 |

## MODEL FORMULATION SYSTEM

The goal of the organization is to determine the quantities to be directed from each crusher to each production unit in order to minimize costs. Thus, the mathematical formulation is as follows:

$$Min\ z = 300x_{11} + 400x_{12} + 550x_{13} + 410x_{21} + 450x_{22} + 580x_{23}$$

S.t

$$x_{11} + x_{12} + x_{13} = 245$$
$$x_{21} + x_{22} + x_{23} = 105$$
$$x_{11} + x_{21} = 100$$
$$x_{12} + x_{22} = 100$$
$$x_{13} + x_{23} = 150$$
$$x_{ij} \geq 0$$

## TRANSPORTATION PROBLEM SOLUTION

We used the simplex method (two-phase) to solve the linear programming model using Excel. Additionally, we applied specialized methods for solving transportation problems (Northwest Corner Method, Least Cost Method, and Vogel's Approximation) both manually and with Excel.

We observed that the results from the simplex method and the specialized transportation methods were identical. We obtained an objective function value (Z) of 155650, and the following constraints:

X11=100: Quantity transported from Al-Aziziyah crusher to the first factory.

X12=100: Quantity transported from Al-Rabita crusher to the first factory.

X13=45: Quantity transported from Bani Walid crusher to the first factory.

X21=0: Quantity transported from Al-Aziziyah crusher to the second factory.

X22=0: Quantity transported from Al-Rabita crusher to the second factory.

X23=105: Quantity transported from Bani Walid crusher to the second factory.

## CONCLUSION

From the practical study we conducted at the Al-Mashat Factory, the primary goal was to reduce costs and determine transportation routes. The study allowed us to demonstrate and confirm the efficiency of the transportation method and its solutions in optimizing decisions related to transporting raw materials (natural stone) from quarries to production units in order to meet the needs of each production center.

***Key Points:***

1. The simplex method can be used to solve linear programming problems for transportation, yielding the same results.
2. The simplex method is highly efficient, but its manual solution process is lengthy and complex.
3. The transportation problem was solved using the three methods (Northwest Corner, Least Cost, and Vogel's Approximation), and the results were consistent with those obtained from the simplex method.

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